Icarus 360 (2021) 114360

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Research paper

Magnetic induction heating of planetary satellites: Analytical formulae and applications

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ARTICLE INFO

Keywords:
Satellites
Dynamics
Magnetic fields
Thermal histories
Jupiter
Io
Europa
Exoplanets

ABSTRACT

Calculations of the magnetic induction heating of planetary satellites to date have been done using approximate or numerical models. Here we present analytical formulae that may be used for either conducting metallic cores (spheres) or conducting water oceans or rock mantles (spherical shells). The formula for the magnetic induction heating of conducting spheres has been in the classic electromagnetic literature for nearly a century, but appears to have been overlooked by astrophysicists. Analogous formulae for conducting spherical shells are derived in this paper. We apply these formulae to calculate induction heating by alternating magnetic fields as seen in the frame of an orbiting satellite. These alternating fields may arise from inclined or horizontally displaced planetary magnetic dipoles, as well as eccentric or inclined satellite orbits. We derive analytical formulae for these, making it easy to investigate quickly the importance of magnetic induction heating for many different system configurations. Magnetic induction heating of planetary satellites in our current Solar System appears unimportant for each of these effects, but may have been greater in the past, in exoplanetary systems, or for other celestial binaries.

1. Introduction

Two forms of electrical heating of celestial objects have been considered in the literature. The first, the homopolar generator, is an astrophysical analog to Lorentz-force-driven current flow and ohmic dissipation in the Faraday disk (Faraday, 1832; Munley, 2004; Chyba et al., 2015). The second is magnetic induction heating due to eddy (Foucault) currents arising from the time-varying magnetic field (as seen in the frame of the secondary) of the primary, according to Faraday's law of induction. These two modes have sometimes been labeled the transverse magnetic (TM) and transverse electric (TE) modes, respectively (Colburn, 1980).

Homopolar induction (TM-mode heating) has been explored as a heating mechanism for Io (Piddington and Drake, 1968; Goldreich and Lynden-Bell, 1969; Colburn, 1980), Europa (Reynolds et al., 1983; Colburn and Reynolds, 1985), Enceladus (Hand et al., 2011), planetesimals by the T-Tauri Sun (Sonnet et al., 1970), and a variety of astrophysical binary systems (see Lai, 2012, for a brief review). In this hypothesis in the case of a planetary satellite, current flows in the ionosphere of the primary, down a flux tube to the primary-facing equatorial region of the satellite, through the conducting satellite, and then back to the primary after exiting the anti-primary equatorial region. The motion of the satellite through the plasma in which it is embedded is analogous to the rotation (in the laboratory frame) of the conducting Faraday disk past the brushes that electrically connect the disk to an external circuit at rest in the laboratory. Possibly the analogy is more obviously made using the linear analog of the homopolar generator (Chyba et al., 2015).

But for each of these satellites, it so far appears likely that the resulting heating is at best minor compared to the total internal heating. At Io, the dense plasma likely shunts the circuit around Io itself, yielding little internal Joule heating (Goertz, 1980; Russell and Huddleston, 2000; Saur et al., 2004). At Europa, the current (and so the heating) is enormously lessened by the resistance of an electrically insulating ice shell; only if the circuit could connect to the conducting ocean through cracks in the ice could there be currents sufficient to generate significant heat (Reynolds et al., 1983). In light of recent observations of possible plumes at Europa (Roth et al., 2014; Sparks et al., 2016), this question needs to be re-examined. At Enceladus, currents may be able to flow through the "tiger stripes" at the south pole, but nevertheless the resulting Joule heating provides less than one percent of the total observed heat flux (Hand et al., 2011).

Magnetic induction (TE-mode heating) has been considered as a potential source of heating for Io (Colburn, 1980), Amalthea (Simonelli,

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https://doi.org/10.1016/j.icarus.2021.114360
Received 16 August 2019; Received in revised form 4 January 2021; Accepted 27 January 2021
Available online 3 February 2021

1983), and Europa's and Callisto's oceans (Khurana et al., 1998), as well as exoplanets close to their stars (Laine et al., 2008). In the Jovian system cases, the satellite experiences a time-varying magnetic field from the primary (Jupiter) due to the tilt of Jupiter's magnetic dipole with respect to the planet's rotation axis. Below we will consider several other sources of time-varying fields as seen in the frame of the satellite. Whatever the source of the oscillation in the magnetic flux density $\mathbf{B}$, these oscillations lead to an electric field $\mathbf{E}$ via the Maxwell–Faraday equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$. Integrating the changing flux over a relevant surface S bounded by a curve C (with element of area $\mathbf{da}$ and line element $\mathbf{dl}$, respectively), and using Stokes' theorem (applicable provided there is no jump discontinuity on S), yields (Giuliani, 2008; Auchmann et al., 2014; Chyba and Hand, 2016) an electromotive force ε given by

$$\varepsilon = \oint_C \mathbf{E} \cdot \mathbf{dl} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{da}. \tag{1}$$

This fluctuating voltage in turn results in eddy currents that dissipate energy in the satellite through ohmic heating.

Colburn (1980) calculated magnetic induction heating for a variety of sub-surface conductivity models for Io using a numerical integration outward from satellite radius $r = 0$ over nested spheres, assigning a small value to $B = |\mathbf{B}|$ at $r = 0$ and scaling the resulting fields to match the external boundary conditions for B. Simonelli (1983) used a whole-body formula stated by Colburn (without citation or derivation) in a table caption for a low-conductivity limit. Khurana et al. (1998) approximated the induction heating in Europa's and Callisto's oceans as analogous to the ohmic loss from a propagating electromagnetic wave in a waveguide. We show below that the approximation adopted by Khurana et al. (1998) is correct if the skin depth in the ocean is small with respect to the thickness of the ocean, but otherwise scales incorrectly with frequency ω and ocean conductivity σ.

In all these cases, recourse to exact analytical formulae is clarifying. At the same time, all these authors correctly concluded that magnetic induction heating is negligible, so there was no need for them to pursue better approximations. The value of the exact formulae comes from their transparency when applied to particular cases and their ease of applicability to cases that have not previously been considered, either within our Solar System or in the realm of exoplanets.

2. Induction heating of conducting spheres

Smythe (1939, 1968) derived an exact analytical formula for the power absorbed by a conducting sphere of uniform conductivity σ and magnetic permeability $\mu = \mu_r \mu_0$ (where $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1} is the permeability of free space and μ_r the relative permeability) in an external sinusoidally alternating magnetic field $\mathbf{H} = \mathbf{B}/\mu_0$, by solving the relevant Poisson equation arising from the Maxwell–Ampère equation for the azimuthal component of the vector potential. (Smythe, 1939 provides this calculation in cgs units; we use here Smythe, 1968, in which calculations are presented in MKS units.) We give Smythe's general result, valid for any μ_r, in Appendix A. Rony (1964) presented a simpler, analogous derivation for $\mu_r = 1$ that of course agrees with Smythe's result for this special case. In this case, the joule heating of a conducting sphere of radius r_0 and conductivity σ in a field $\mathbf{B}$ oscillating with angular frequency ω is

$$P_{\text{sphere}} = \frac{3\pi r_0 B^2}{\mu_0^2 \sigma} G(x), \tag{2}$$

where

$$G(x) = \frac{x(\sinh 2x + \sin 2x)}{\cosh 2x - \cos 2x} - 1, \tag{3}$$

$x = r_0/\delta$, and we emphasize that Eq. (2) holds only for the special case $\mu_r = 1$. The skin depth δ gives the e-folding depth into the conductor for the diffusion of the alternating field $\mathbf{B}$; it is in general

$$\delta = (2/\mu\omega\sigma)^{1/2}, \tag{4}$$

which becomes $\delta = (2/\mu_0\omega\sigma)^{1/2}$ for $\mu_r = 1$.

Setting $\mu_r = 1$ is the right choice for rock or ice, but could be questioned if we were considering, say, the iron core of a sufficiently small satellite. But even here $\mu_r = 1$ is still likely the correct approximation since $\mu_r = 1$ if the temperature of the core is above the Curie temperature T_c of iron. For iron $T_c = 1043$ K with little pressure dependence, rising to only about 1053 K at the boundary of Earth's outer core (Campbell, 2003), where the pressure is around 140 GPa. Were we interested in iron cores below T_c, however, Smythe's (1968) full formula could be employed. In this case, four additional terms enter into the denominator of the expression for joule heating. We show in Appendix A that for parameter choices relevant to planetary satellites, these terms sum to a positive quantity, thereby *decreasing* ohmic heating below that given by Eq. (2), which assumes $\mu_r = 1$. Since heating will prove negligible when $\mu_r = 1$, we do not consider the case $\mu_r \neq 1$ further here.

We can then proceed to examine physically relevant limiting cases of Eqs. (2) and (3). When $\delta \ll r_0$ ($x \gg 1$), e.g. for σ sufficiently large, Eq. (3) becomes $G(x) = x = r_0/\delta$ and Eq. (2) yields (Fromm and Jehn, 1965):

$$P_{\text{sphere}}^{\delta \ll r_0} = \frac{3\pi\omega^{1/2} B^2 r_0^2}{\sqrt{2}\mu_0^{3/2}\sigma^{1/2}}, \tag{5}$$

where the superscript indicates the assumption made to derive Eq. (5) for P_{sphere}. By Eq. (5), joule heating decreases as σ increases. Physically this can be understood by considering the volume distribution of heating in the sphere. When $\delta \ll r_0$, we can approximate $\mathbf{B}$ as constant over the skin depth and zero elsewhere in the sphere; this approximation is good to $\delta/2r_0$. Therefore the heating of the sphere occurs only in its outermost layer of volume $4\pi r_0^2\delta$. The volumetric heating in this outermost layer is then, from Eq. (5),

$$\frac{P_{\text{sphere}}^{\delta \ll r_0}}{\text{heated volume}} = \frac{3}{8}\frac{\omega B^2}{\mu_0}. \tag{6}$$

This qualitatively agrees with, but is smaller by a factor of 3 than, the limit for large σ that Colburn (1980) found in his numerical investigations. Eq. (6) shows that the volumetric heating of the heated volume $4\pi r_0^2\delta$ becomes independent of σ as the skin depth gets small. But the heated volume itself gets smaller according to Eq. (4) as σ increases, so P_{sphere} must decrease.

Now consider the limit $\delta \gg r_0$ ($x \ll 1$) that might apply for a world of low conductivity, where the oscillating magnetic field penetrates the entire satellite with minimal attenuation. In this limit, Eq. (3) becomes $G(x) = x^4/15$, where it is necessary to keep sixth-order terms in the expansions for the hyperbolic and trigonometric functions in $G(x)$. Eq. (2) becomes

$$P_{\text{sphere}}^{\delta \gg r_0} = (\pi/15)\sigma\omega^2 B^2 r_0^5, \tag{7}$$

so that heating is proportional to σ when σ is sufficiently small. Then

$$\frac{P_{\text{sphere}}^{\delta \gg r_0}}{\text{heated volume}} = \frac{3}{20}\sigma\omega^2 B^2 r_0^2, \tag{8}$$

where the volume of the heated sphere is just $(4/3)\pi r_0^3$. Eq. (8) is within a factor of 2 of the limit Colburn (1980) stated without derivation for small σ. It is identical to within a numerical coefficient with the expression for the induction heating of a bar of square cross section d^2 where d is small compared to the skin depth (Williams et al., 1950; O'Handley, 2000).

Eqs. (2), (5) and (7) suggest that induction heating of conducting spheres is not significant across any reasonable range of satellite parameters in our contemporary Solar System, given the strength of planetary magnetic fields. As an illustration, consider a uniformly conducting sphere in the orbit of Io and let σ run from, say, 10^{-6} S m^{-1} to 10^7 S m^{-1} (the latter value being about that of the conductivity of pure iron). Consistent with Io's orbit around Jupiter (with a magnetic dipole

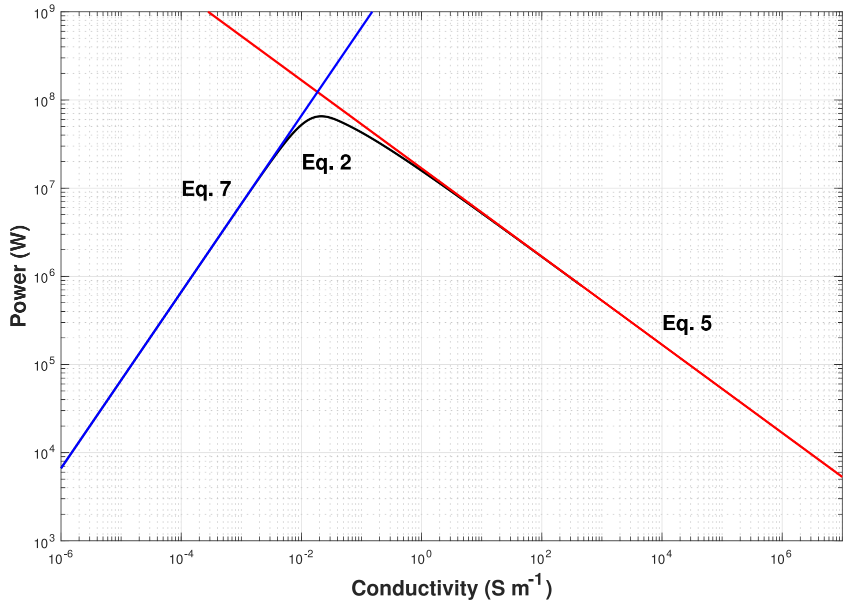

Fig. 1. Induction heating of Io, treated as a sphere of uniform conductivity, from Eq. (2) for a range of conductivities, with comparisons for the limiting cases of Eqs. (5) and (7). For any conductivity, heating appears unimportant.

inclined by 9.6°), we take $\omega = 1.3 \times 10^{-4}$ s^{-1} (Colburn, 1980), the difference between Jupiter's rotation frequency and Io's mean motion. (Recall that we are working in Io's frame, so that this difference is the apparent frequency of Jupiter's rotating inclined dipole.) We also have $B = 1835\sin(9.6°)$ nT $= 306$ nT (Kivelson et al., 1996), and $r_0 = 1821$ km for Io. For specificity, consider Eq. (5) for $\sigma = 1 \times 10^6$ S m^{-1}, the conductivity appropriate to FeS at a temperature of 1900 K and a pressure of 6 GPa (Li et al., 2007). With these values, $P_{\text{sphere}} = 17$ kW, a stunningly small number. (This may be compared with the original calculation of tidal heating for Io that gave 1600 GW for a dissipation factor Q = 100 for Io; Peale et al., 1979.) To test sensitivity to a different choice of σ, in Fig. 1 we display Eq. (2) for these parameters as a function of σ; maximum P_{sphere} across this range is only about 60 MW, an insignificant amount of heating. Of course, the actual satellite Io has a non-uniform interior structure with different conductivities appropriate to its various regions (Khurana et al., 2011). However, since Eq. (5) scales like r_0^2, treating Io as a sphere of radius $r_0 = 1821$ km will, for a given σ, maximize estimates of induction heating. Since even these cases (Fig. 1) yield a negligible result, we can be confident that a more realistic treatment would not change this conclusion.

3. Induction heating of spherical shells

A variety of planetary satellite features are better modeled by spherical shells than by complete spheres, for example conducting water oceans. If the conducting spherical shell is underlain by a layer of much lower conductivity, the spherical shell model should apply. Even if the conducting shell is underlain by another strongly conducting material, a shell model will be applicable if the thickness h of the shell satisfies $h \gg \delta$, where δ is the skin depth in the shell. Here we outline our approach for deriving formulae for the induction heating of spherical shells, displaying key results; the detailed calculations are performed in Appendix B.

The power P absorbed in a conductor is given by

$$P = I^2 R = \frac{\varepsilon^2 R}{Z^2}, \tag{9}$$

where ε is the emf induced in the conductor (in our case due to a flux density **B** oscillating at angular frequency ω), the current $I = \varepsilon / Z$, and squares are understood to mean the square modulus, e.g. $\varepsilon^2 = \varepsilon^* \varepsilon$. Here $Z = R + i\omega L$ is the conductor's impedance, with square modulus

$$Z^2 = R^2 + (\omega L)^2, \tag{10}$$

with R and L the conductor's resistance and inductance, respectively.

To use Eq. (9) to calculate P_{shell}, the power absorbed in a conducting shell, we therefore need $\varepsilon_{\text{shell}}$, R_{shell}, and Z_{shell}. The latter in turn requires L_{shell}. The shell's resistance R_{shell} is readily calculated, but calculating L_{shell} is more challenging. The calculation is not given in standard compilations of inductances for different objects and geometries (Rosa and Grover, 1948; Cohen, 1996; Grover, 2009). Perhaps the closest calculation in the literature of which we are aware is for the inductance of a spherical solenoid (Wheeler, 1958), but of course for a solenoid issues of skin depth are irrelevant, so the inductance of the spherical solenoid cannot be appropriate for a conducting spherical shell. We calculate R_{shell}, Z_{shell}, and L_{shell} in Appendix B.

Results for R_{shell} and L_{shell} depend strongly on whether the shell (of thickness h, say) is thick ($h \gg \delta$) or thin ($h \ll \delta$) compared to the skin depth δ. On physical grounds we expect the thick shell result to be identical to that for a solid sphere, Eq. (5), and this serves as a check on our approach.

For a thick ($h \gg \delta$) shell of radius r_0 and thickness h, we find (Appendix B)

$$R_{\text{shell}}^{\delta \ll h} = \frac{\pi}{2\sigma\delta}, \tag{11}$$

$$L_{\text{shell}}^{\delta \ll h} = \frac{\sqrt{3}\pi}{4}\mu_0 \delta, \tag{12}$$

and

$$Z_{\text{sphere}}^2 = \frac{\pi^2 \omega \mu_0}{2\sigma} = 4R_{\text{sphere}}^2, \tag{13}$$

where we have dropped the superscript $\delta \ll h$ to avoid confusion with the exponent. In addition, we find

$$\varepsilon^2 = (3/2)\pi^2 \omega^2 B^2 r_0^2 \delta^2, \tag{14}$$

so that by Eq. (9),

$$P_{\text{shell}}^{\delta \ll h} = \frac{3\pi \omega^{1/2} r_0^2 B^2}{\sqrt{2}\mu_0^{3/2} \sigma^{1/2}} \tag{15}$$

for the thick conducting spherical shell, results identical to those for the conducting sphere for the case $\delta \ll r_0$ (e.g., Eq. (5)).

For a thin ($h \ll \delta$) shell of radius r_0 and thickness h, we find (Appendix B)

$$R_{\text{shell}}^{\delta \gg h} = \frac{\pi}{2\sigma h}, \tag{16}$$

$$L_{\text{shell}}^{\delta \gg h} = \frac{\sqrt{3}\pi}{8}\mu_0 r_0, \tag{17}$$

and

$$\varepsilon^2 = (\pi^2/2)\omega^2 B^2 r_0^4, \tag{18}$$

so that by Eq. (9),

$$P_{\text{shell}}^{\delta \gg h} = \pi\sigma\omega^2 B^2 r_0^4 h \left(1 + \frac{3r_0^2 h^2}{4\delta^4}\right)^{-1}. \tag{19}$$

If the impedance of the shell is dominated by the resistance (e.g. for sufficiently low conductivities), Eq. (19) becomes

$$P_{\text{shell}}^{\delta \gg h} = \pi\sigma\omega^2 B^2 r_0^4 h. \tag{20}$$

If the impedance of the shell is instead dominated by the inductive reactance (e.g. for very high conductivities), Eq. (19) becomes

$$P_{\text{shell}}^{\delta \gg h} = \frac{16\pi}{3}\frac{B^2 r_0^2}{\mu_0^2 \sigma h}. \tag{21}$$

4. Planetary dipoles and satellite orbits

Seen from an orbiting satellite, a tilted planetary dipole results in a magnetic field varying sinusoidally along the radial line from the planet's center to the satellite. The magnitude of the effect is proportional to the magnitude of the inclination of the dipole axis. For an inclination angle θ, this gives an alternating component of B equal to $B\sin\theta$. For example, $\theta = 9.6°$ for Jupiter but $0°$ for Saturn. Here we consider the effect of tilted dipoles but also other sources of time-varying **B**, and investigate under what conditions these might provide quantitatively more important sources of induction heating.

Tilted planetary dipoles. Consider applying our results to the Europan ocean, taken here to be a conducting spherical shell of thickness $h = 100$ km. For Europa, $\omega = 1.6 \times 10^{-4}$ s^{-1}, $B = 220$ nT for the peak equatorial amplitude of Jupiter's oscillating field (Khurana et al., 1998) and $r_0 = 1560$ km. The skin depth of B into the ocean, by Eq. (4), is then

$$\delta = 100 \text{ km} \left(\frac{1 \text{ S m}^{-1}}{\sigma}\right)^{1/2}, \tag{22}$$

Even if Europa's ocean were salt-saturated, σ would be less than 20 S m^{-1} (Hand and Chyba, 2007). For $\sigma = 10$ S m^{-1}, $\delta = 32$ km, which we will take as just satisfying the requirement $\delta \ll h$ necessary for Eq. (15) to apply. We find $P_{shell} = 7$ MW, consistent with the results found by Khurana et al. (1998) and quite negligible.

Could heating be increased for oceans of much lower conductivity? Consider instead $\sigma = 10^{-1}$ S m^{-1}, for which $\delta = 1.0 \times 10^3$ km, which satisfies $\delta \gg h$. In this case Eq. (19) applies. In fact, the resistance term in the denominator of Eq. (19) dominates the inductive reactance term by two orders of magnitude in this case, so we may use Eq. (20), and we find $P_{shell} = 330$ kW, again a negligible amount of heating. Decreasing σ in Eq. (20) will, of course, only make P_{shell} smaller still.

Horizontally offset planetary dipoles. Seen from an orbiting satellite, a tilted planetary dipole results in a magnetic field varying sinusoidally along the radial line from the primary's magnetic dipole to the satellite. Planetary dipoles may also appear to be horizontally offset from the planet's rotation axis; for example, Jupiter's dipole may be modeled as an tilted dipole that is horizontally offset by a distance $R_h = 0.13R_J$ (NSSDC, 2014). Since dipole magnetic fields fall off with the cube of the distance, this means that an orbiting satellite will experience a component of the planet's magnetic field normal to the orbit of the satellite (taking the satellite to orbit in the planet's equatorial plane) that varies sinusoidally with the sidereal period. If the magnitude of the planet's magnetic flux density normal to the satellite's orbit is B, the amplitude of the sinusoidal variation is $3(R_h/a)$, where a is the satellite's semi-major axis, and we have assumed $R_h \ll a$. That is, for a horizontal offset R_h, this gives an alternating component of B equal to $3(R_h/a)B$.

Consider for specificity this effect at Jupiter's moon Io. With $R_J =$ 71,398 km and $a = 5.91R_J$ (NSSDC, 2014), this gives an alternating component of B equal to $3(R_h/a)B = 0.066(1835 \text{ nT}) = 121$ nT. The magnitude of this effect is a bit more than a third as great as that due to Jupiter's dipole's tilt, which yields $(1835 \text{ nT})\sin 9.6° = 306$ nT, with eddy currents that run perpendicular to those set up by the dipole's tilt.

Eccentric satellite orbits. As a satellite travels in an orbit with eccentricity e, it periodically moves from apoapse to periapse, moving from a maximum distance $(1+e)a$ to a minimum distance $(1-e)a$ from its primary. Since the magnetic field strength falls off with the cube of the distance, we can approximate this effect (as seen in the frame of the satellite) as a magnetic field that is fluctuating with amplitude $3e$ about its value at a distance a from the primary. That is, for example, the difference in magnetic field strength at the apoapse distance $(1+e)a$ compared to the average distance a is just $a^{-3} - [(1+e)a]^{-3} \approx 3ea^{-3}$, provided $e \ll 1$. For a satellite orbit with eccentricity e, this gives an alternating component of B equal to $3eB$. For the example of Io, for which e=0.004 (NSSDC, 2014), $3eB = 22$ nT. Heating therefore scales proportionally to e^2. The relevant frequency for this case (and the inclination case below) is just the orbital mean motion n.

Inclined satellite orbits. A magnetic dipole varies in the polar angle direction like $\sin\theta$. A satellite in a near-equatorial orbit with a small inclination i will therefore experience, around its orbit, an alternating component of B that varies from $B\sin(\pi/2-i) = B\cos(-i) = B - B(i^2/2)$ to B around a quarter of an orbit. However, $i^2/2$ for typical satellite orbits is very small; e.g. for Io with $i = 0.04°$, $i^2/2 = 2 \times 10^{-7}$, and (barring satellites with high-inclination orbits) the effect is negligible.

5. Conclusion

Magnetic induction heating of planetary satellites may occur due to inclined or horizontally offset planetary dipoles, or due to eccentric or inclined satellite orbits. Induction heating of conducting spheres or spherical shells may be evaluated using analytical formulae that come out of the classic electromagnetism literature (spheres) or are derived here (spherical shells). In the case of highly conductive spheres or spherical shells, volumetric heating is limited due to the oscillating magnetic field's limited penetration into the conductor. In the case of a low conductivity conductor, the inductive reactance becomes very large. For the effects considered here, magnetic induction heating appears to be negligible for planetary satellites in our current Solar System.

However, we have provided formulae that can be applied either to earlier periods in Solar System history, to exoplanet systems, or to other celestial binaries, where it is possible for induction heating to be of greater significance—though in fact, this remains difficult to achieve. As an illustration, consider a Europa-like moon of radius 2000 km in orbit around a Jupiter-like primary of rotation period 10 h. Endow the moon with a conducting ocean of thickness $h = 10$ km and $\sigma = 1$ S/m, experiencing a time-varying (due to dipole tilt) magnetic field of 2×10^{-5} T. Such a field can be achieved for a Jupiter-like primary provided that the moon is orbiting closer than Io, and/or the primary's magnetic dipole is tilted more than that of Jupiter, and/or the primary has a larger intrinsic magnetic field than that of Jupiter. For these choices, by Eq. (19) the satellite's ocean would experience about 1 TW of induction heating. A remarkable property of induction heating due to tilted or offset planetary dipoles is that the effect is independent of the eccentricity or inclination of the satellite orbit, so that circularization of the orbit will not affect the heating. This of course is quite distinct from the strong dependence (e.g., Chyba et al., 1989) of tidal heating on orbital eccentricity and inclination.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors thank R. Reynolds, D. S. Colburn, F. Thomson, and R. V. E. Lovelace for past conversations over many years on the broad subject of this paper. KPH acknowledges support from the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. PJT thanks the Office of Research and Sponsored Programs at the University of Wisconsin-Eau Claire for sabbatical support.

Appendix A. Induction heating for a conducting sphere of arbitrary magnetic permeability

Smythe (1968) derives a formula for the joule heating of a conducting sphere of arbitrary magnetic permeability. (See Smythe, 1939, for this formula in cgs units.) He finds:

$$P_{\text{sphere}} = \frac{3\pi\sigma\mu_r^2\mu_0^2\omega^2 r_0^5 B^2\{x[\sinh(2x)+\sin(2x)]-\cosh(2x)+\cos(2x)\}}{(\mu_r-1)^2\mu_0^2 M + (\mu_r-1)\mu_0 N + 4\mu_0^2 x^4[\cosh(2x)-\cos(2x)]}, \tag{A.1}$$

where

$$M = (2x^2+1)\cosh(2x) + (2x^2-1)\cos(2x) - 2x[\sinh(2x)+\sin(2x)], \tag{A.2}$$

$$N = 4\mu_0 x^3[\sinh(2x)-\sin(2x)], \tag{A.3}$$

$x = r_0/\delta$, and δ is given by Eq. (4). When $\mu_r = 1$, Eq. (A.1) collapses to Eq. (2).

Now consider the case $\mu_r \neq 1$ in Eq. (A.1). In the context of planetary satellites, this might hold for, say, an iron core that somehow remained below the Curie temperature. In any such case $\mu_r \gg 1$, and μ and σ contribute to making δ in Eq. (4) very small compared with any plausible value of r_0. We therefore have $x \gg 1$, and Eq. (A.1) becomes

$$P_{\text{sphere}} = \frac{3\pi\sigma\mu_r^2\mu_0^2\omega^2 r_0^5 B^2}{2\mu_r^2\mu_0^2 x + 4\mu_r\mu_0^2 x^2 + 4\mu_0^2 x^3}, \tag{A.4}$$

where we have also used $\mu_r \gg 1$. Using $x = r_0/\delta$ and Eq. (4), we see that if only the third term in the denominator were present, Eq. (A.4) would be larger than Eq. (5) by a factor $\sqrt{\mu_r}$, which might be $\sim 10^2$ for an iron core. In this case P_{sphere} remains negligible. The remaining two terms in the denominator in Eq. (A.4) are both positive, so can only decrease P_{sphere} further compared to this estimate. Therefore even if we were to have a world where an iron core were below the Curie temperature, this still would not lead to significant magnetic induction heating of that core.

Appendix B. Calculations for spherical shells

Inductance L is defined by the relation

$$\Phi = LI, \tag{B.1}$$

where the magnetic flux through a surface S with area element $\mathbf{da}$ is

$$\Phi = \int_S \mathbf{B}\cdot\mathbf{da} \tag{B.2}$$

and I is the total current. For identical currents, the inductance $L = \Phi/I$ for a spherical shell differs from that for the solid sphere by the ratio of the areas penetrated by $\mathbf{B}$ that contribute to the integral in Eq. (B.2). Our approach will be to calculate L_{sphere} using Eq. (2) and then to determine L_{shell} from L_{sphere} by taking the appropriate area ratio.

First, then, we calculate L_{sphere} – a result that, like that for a spherical shell, is not to be found in the compilations of inductances for different objects and geometries (Rosa and Grover, 1948; Grover, 2009). We begin by calculating the emf for the sphere. Once we have the emf, we calculate the sphere's resistance and use Eqs. (9) and (10) to solve for L_{sphere} in terms of $\varepsilon_{\text{sphere}}$, R_{sphere}, and P_{sphere}. We know P_{sphere} from Eq. (2), or Eqs. (5) and (7) in the appropriate limits.

Consider a magnetic flux density

$$\mathbf{B} = Be^{-i\omega t}\hat{\mathbf{z}} \tag{B.3}$$

incident on a conducting sphere of radius r_0, conductivity σ and magnetic permeability μ_0. We use the usual spherical coordinate (r,θ,φ) and cylindrical coordinate (ρ,ϕ,z) definitions and relations. The sinusoidal oscillations drive alternating currents that run azimuthally around the sphere. The flux density $\mathbf{B}$ falls off exponentially into the sphere with a skin depth given by Eq. (4). We first calculate the emf for one azimuthal ring at colatitude θ in the limit $\delta \ll r_0$. Eq. (1) gives:

$$\varepsilon_{\text{ring}} = -i\omega\int_0^{2\pi}\int_{r_0\sin\theta}^{0} Be^{-i\omega t}e^{-(r_0-r)/\delta}\rho\, d\phi\, d\rho = -2\pi i\omega Be^{-i\omega t} r_0\delta\sin^2\theta, \tag{B.4}$$

where we have used $r = \rho/\sin\theta$ to evaluate the integral. This same result can be obtained from Eq. (1) by making the approximation that $\mathbf{B}$ is uniform across the outer layer δ of the sphere and $\mathbf{0}$ within (Wouch and Lord, 1978). Then

$$\varepsilon^2_{\text{sphere}} = \varepsilon_{\text{ring}}\varepsilon^*_{\text{ring}} = \frac{3}{2}\pi^2\omega^2 r_0^2\delta^2 B^2 = \frac{3\pi^2\omega r_0^2 B^2}{\mu_0\sigma}, \tag{B.5}$$

where we have averaged over θ over the entire sphere using $(1/\pi)\int_0^\pi \sin^4\theta\, d\theta = 3/8$ and for the final equality used Eq. (4) with $\mu = \mu_0$.

The resistance R_{sphere} may be calculated from the definition for resistance

$$R = \oint \frac{dl}{\sigma A}, \tag{B.6}$$

where dl is the current differential path length and A the cross-sectional area of that path. To calculate R_{sphere} for azimuthal currents running about the $\delta \ll r_0$ sphere, we put $A = \pi r_0\delta$, and calculate the average circumference of an azimuthal current path to be

$$\bar{C} = \frac{1}{r_0}\oint_0^{r_0} 2\pi\rho\, dz = \frac{1}{r_0}\oint_0^{r_0} 2\pi(r^2-z^2)^{1/2}dz = \frac{\pi^2 r_0}{2}. \tag{B.7}$$

Then Eq. (B.6) gives

$$R^{\delta\ll r_0}_{\text{sphere}} = \frac{\bar{C}}{\sigma A} = \frac{\pi}{2\sigma\delta} = \frac{\pi}{2}\left(\frac{\omega\mu_0}{2\sigma}\right)^{1/2}. \tag{B.8}$$

Equating Eqs. (5) and (9) using Eqs. (4), (B.5) and (B.8) then gives for the impedance of the $\delta \ll r_0$ sphere

$$Z^2_{\text{sphere}} = \frac{\pi^2\omega\mu_0}{2\sigma} = 4R^2_{\text{sphere}}, \tag{B.9}$$

or $|Z_{\text{sphere}}| = 2R_{\text{sphere}}$. Finally, using Eqs. (10), (B.8) and (B.9) to solve for L_{sphere}, we find

$$L^{\delta\ll r_0}_{\text{sphere}} = \sqrt{3}\frac{R^{\delta\ll r_0}_{\text{sphere}}}{\omega} = \frac{\sqrt{3}\pi}{4}\mu_0\delta = \frac{\pi}{2}\left(\frac{3\mu_0}{2\omega\sigma}\right)^{1/2}. \tag{B.10}$$

Since $\omega L^{\delta\ll r_0}_{\text{sphere}} = \sqrt{3}R^{\delta\ll r_0}_{\text{sphere}}$, the impedance of the solid conducting sphere is slightly dominated by its inductive reactance, but the resistance and inductive reactance are comparable in magnitude. This explains why Wouch and Lord (1978) were able to come within a small factor of the correct answer for inductive power dissipation in a $\delta \ll r_0$ conducting sphere, Eq. (5), despite ignoring the inductive reactance of the sphere.

We now determine L_{shell} from L_{sphere} by taking area ratios. For spherical shells of radius r_0 with thickness h, we consider thick ($h \gg \delta$) and thin ($h \ll \delta$) shells. The relevant areas threaded by the magnetic field are identical for the $\delta \ll r_0$ sphere and the $\delta \ll h$ thick shell, so the inductance for the thick shell is just

$$L_{\mathrm{shell}}^{\delta \ll h} = \frac{\sqrt{3}\pi}{4}\mu_0 \delta. \tag{B.11}$$

Similarly, it is clear that the calculations for R, Z, and ε are identical in the two cases. Therefore $P_{\mathrm{shell}}^{\delta \ll h} = P_{\mathrm{sphere}}^{\delta \ll r_0}$ as well.

The inductance for the thin ($\delta \gg h$) shell is greater than that for the solid sphere by the ratio of the areas penetrated by $\mathbf{B}$, that is, by $\pi r_0^2/2\pi r_0 \delta = r_0/2\delta$. Then, by Eq. (B.11),

$$L_{\mathrm{shell}}^{\delta \gg h} = \frac{\sqrt{3}\pi}{8}\mu_0 r_0. \tag{B.12}$$

Calculating the resistance of the thin shell from Eq. (B.6) proceeds as for the $\delta \ll r_0$ sphere or for the thick shell case except that now the cross-sectional area $A = \pi r_0 h$. We therefore have

$$R_{\mathrm{shell}}^{\delta \gg h} = \frac{\pi}{2\sigma h}. \tag{B.13}$$

so

$$\frac{R_{\mathrm{shell}}^{\delta \gg h}}{\omega L_{\mathrm{shell}}^{\delta \gg h}} = \frac{2\delta^2}{\sqrt{3} r_0 h}. \tag{B.14}$$

By Eqs. (10) and (B.14), whether the impedance for the shell is dominated by the resistance or the inductive reactance depends on how $\delta^2/r_0 h$ compares to 1. By assumption for the thin shell, $\delta/h \gg 1$, but we could simultaneously have $\delta/r_0 \ll 1$, so the answer is unclear and will depend on the details of the system. From Eqs. (10), (B.12) and (B.13),

$$(Z_{\mathrm{shell}}^{\delta \gg h})^2 = \left[\left(\frac{\pi}{2\sigma h}\right)^2 + 3\left(\frac{\pi r_0}{4\sigma \delta^2}\right)^2\right]. \tag{B.15}$$

Calculating the emf for the thin shell proceeds analogously to the calculation for Eq. (B.4), so that

$$\varepsilon_{\mathrm{ring}} = -i\omega \int_0^{2\pi} \int_{r_0 \sin\theta}^{0} B e^{-i\omega t} \rho \, d\phi \, d\rho = -\pi i \omega B e^{-i\omega t} r_0^2 \sin^2\theta, \tag{B.16}$$

and

$$\varepsilon_{\mathrm{ring}}^2 = (\pi^2/2)\omega^2 r_0^4 B^2. \tag{B.17}$$

By Eqs. (9), (B.13) and (B.15),

$$P_{\mathrm{shell}}^{\delta \gg h} = \pi \sigma \omega^2 B^2 r_0^4 h \left(1 + \frac{3 r_0^2 h^2}{4\delta^4}\right)^{-1}. \tag{B.18}$$

If the impedance of the shell is dominated by the resistance, Eq. (B.18) becomes

$$P_{\mathrm{shell}}^{\delta \gg h} = \pi \sigma \omega^2 B^2 r_0^4 h. \tag{B.19}$$

If the impedance of the shell is instead dominated by the inductive reactance, Eq. (B.18) becomes

$$P_{\mathrm{shell}}^{\delta \gg h} = \frac{16\pi}{3} \frac{B^2 r_0^2}{\mu_0^2 \sigma h}. \tag{B.20}$$

In this limit, the power dissipated in the shell is independent of ω, a surprising result. However, only values of ω satisfying two simultaneous constraints are consistent with Eq. (B.20): ω must fulfill the "thin shell" requirement $h \ll \delta$ as well as the condition $(\omega L_{\mathrm{shell}})^2 \gg R_{\mathrm{shell}}^2$. Requiring these two constrains simultaneously and writing the period of oscillation of $\mathbf{B}$ as $T = 2\pi/\omega$ leads to

$$\pi \tau_D \ll T \ll \frac{\sqrt{3}\pi}{2} \frac{r_0}{h} \tau_D, \tag{B.21}$$

where

$$\tau_D = \sigma \mu_0 h^2 \tag{B.22}$$

is just the characteristic diffusion time required for the magnetic flux to diffuse into a shell of thickness h. If the oscillation period of $\mathbf{B}$ is much longer than the diffusion time into the shell and much shorter than the diffusion time scaled by the factor r_0/h, induction heating is independent of ω, a result that could be of interest in laboratory induction heating of metals (Rony, 1964; Wouch and Lord, 1978).

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